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THERMOELASTIC STRESSES IN COMPOSITE CERAMIC FIBERS

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A calculation of stress and deformation fields in ceramic fibers formed by the method of chemical vapor deposition onto a heated substrate is performed within the framework of linear elasticity theory. Optimum parameters for fibers with a homogeneous structure, a layered structure, and a gradient one are sought.

In the process of forming fibers by the method of chemical vapor deposition onto a heated substrate from a mixture of reactive gases, both product layers that are homogeneous in composition and structure and that are inhomogeneous (multilayer fibers, gradient fibers, etc.) may form. At temperatures that are different from the conditions of synthesis for fibers fields of stresses and deformations arise in the latter, of which tensile stresses are the most "hazardous" to ceramic materials. In the present work we state and solve the problem of calculating thermal stresses in homogeneous, laminated, and gradient fibers within the framework of linear elasticity theory and seek conditions under which tensile stresses become minimum. The results obtained can be used to optimize the phase composition and structure of fibers, which is of great practical interest in developing composite materials of a new generation.

1. Statement of the Problem. Current methods for calculating stress-strain states in composite materials are usually based on the hypothesis of uniform deformation for all components of the composite [1–4] or on using ideas of the so-called "close relationship" between the composite elements (equality of components of the displacement vector and the corresponding components of the stress tensor on interfaces between the elements) [5, 6]. The present work deals with the case of a spatial distribution of the thermal expansion coefficient (TEC) while the remaining parameters of the system are considered as equal. The initial specimen is a thread (cylinder) with the characteristic dimension R, onto which a product layer of thickness δ is deposited at temperature T_0. Upon deposition of the coating the temperature of the thread-product layer system changed slowly to some T. By assuming that in the absence of external forces when T_0 is prescribed the fiber is in the underformed state and the difference $(T-T_0)$ is small we find the relationship between the field of the stresses p_{ik} that emerge in the system at temperature T and the characteristics of the coating for different cases of the dependence $\alpha[C(r)]$ where C is some parameter, for example, the concentration of a deficient reagent, governing the TEC of the coating layer at point r.

The assumptions made permit the use of the expression for the compound energy of a body deformed with change in temperature [7]:

$$F(T) = F(T_0) - k\alpha\,[C]\,(T - T_0)\,u_{ll} + \mu\left(u_{ik} - \frac{1}{3}\delta_{ik}\,u_{ll}\right)^2 + \frac{k}{2}u_{ll}^2,$$

whence, in view of the equilibrium conditions $(\partial p_{ik}/\partial x_k) = 0$, where $p_{ik} = (\partial F/\partial u_{ik})$, we can obtain the equation for deformations

$$3\,\frac{1-\sigma}{1+\sigma}\,\mathrm{grad}\,\mathrm{div}\,\overline{u} - \frac{3}{2}\,\frac{1-2\sigma}{1+2\sigma}\,\mathrm{rot}\,\mathrm{rot}\,\overline{u} = (T - T_0)\,\mathrm{grad}\,\alpha\,, \tag{1}$$

which together with the condition of finite deformations at the center of symmetry

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$$\bar{u}|_{r=0} < \infty \tag{2}$$

and the condition of vanishing radial stresses on the free surface of the coating

$$p_{rr}|_{r=R+\delta} = 0 \tag{3}$$

governs the mathematical formulation of the stated problem.

Thus, we have come to problem (1)-(3), which is identical in mathematical aspect to the problem of equilibrium of nonuniformly heated isotropic solids (see [7, pp. 35 and 36]). However, the equilibrium conditions for heated (cooled) but no longer homogeneous (with respect to α) solid systems are dealt with physically here.

2. Results of the Calculations. Due to the symmetry of problem (1)-(3) ($\bar{u} \,||\, \bar{r}$, $\bar{u} = \bar{u}(r)$, i.e., rot $\bar{u} = 0$), we can reduce Eq. (1) to the simpler form

$$\frac{d}{dr}\left(\frac{1}{r^n}\frac{d(r^n u)}{dr}\right) = \frac{1}{3}\frac{1+\sigma}{1-\sigma}(T - T_0)\frac{d\alpha}{dr}. \tag{4}$$

Here $n = 0, 1, 2$ is plane, cylindrical, and spherical symmetry. Integrating Eq. (4) with boundary conditions (2) and (3) yields in the case of cylindrical symmetry ($n = 1$)

$$u = \frac{(1+\sigma)(T-T_0)}{3(1-\sigma)}\left\{\frac{1}{r}\int_0^r \alpha r dr + \frac{C_1 r}{2}\right\}, \tag{5}$$

$$u_{rr} = \frac{(1+\sigma)(T-T_0)}{3(1-\sigma)}\left\{\alpha - \frac{1}{r^2}\int_0^r \alpha dr + \frac{C_1}{2}\right\}, \tag{6}$$

$$u_{\varphi\varphi} = \frac{1}{r}u, \quad u_{zz} = 0, \quad u_{ik}|_{i \neq k} = 0,$$

$$\frac{C_1}{2} = (1-2\sigma)\frac{1}{(R+\delta)^2}\int_0^{R+\delta} \alpha r dr - (1-\sigma)\,\alpha|_{R+\delta}, \tag{7}$$

where u_{ik} is the deformation tensor and there is a linear relationship between u_{ik} and the stress tensor p_{ik}:

$$p_{rr} = \frac{E}{(1+\sigma)(1-2\sigma)}[(1-\sigma)u_{rr} + \sigma(u_{\varphi\varphi} + u_{zz})], \tag{8}$$

$$p_{ik} = \frac{E}{(1+\sigma)(1-2\sigma)}[(1-2\sigma)u_{ik} + \sigma u_{ll}\delta_{ik}].$$

Homogeneous Fibers. We consider a step distribution of the TEC, conforming to this case, in a cylindrically symmetric system:

$$\begin{aligned} &\alpha = \alpha_0 \quad \text{for} \quad 0 < r < R; \\ &\alpha = \alpha_c \quad \text{for} \quad r > R. \end{aligned} \tag{9}$$

The deformation and stress fields (see (5)-(8)) that correspond to the distribution (9) have the form

$$\omega = \begin{cases} \xi(1-\nu)\,[1 + (1-2\sigma)R^2/(R+\delta)^2], & \xi < 1; \\ (1-\nu)\,[1/\xi + (1-2\sigma)R^2\xi/(R+\delta)^2], & \xi > 1, \end{cases} \tag{10}$$

where

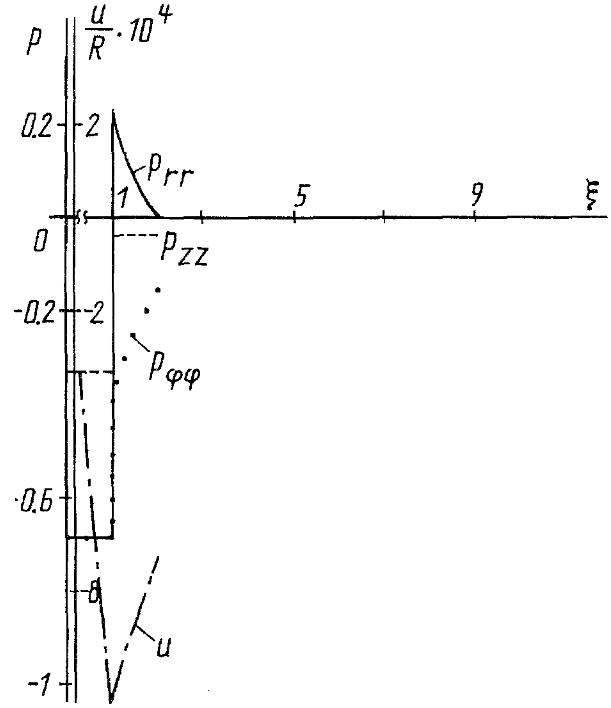

Fig. 1. Deformation and stress fields in a W–B system: $\alpha(\mathrm{W}) = \alpha_0 \equiv 4.6 \cdot 10^{-6}\ \mathrm{K}^{-1}$, $\alpha(\beta) = \alpha_c = 1.1 \cdot 10^{-6}\ \mathrm{K}^{-1}$, $E(\mathrm{W}) \simeq E(\mathrm{B}) = E \equiv 410$ HPa, $\sigma = 0.24$, $T - T_0 = -10^3$ K, $\delta/R = 1$. p, HPa.

$$\omega = 6\,(1 - \sigma)\, u/(1 + \sigma)\, \alpha_0 R\, (T - T_0)\,, \quad \xi = r/R\,, \quad \nu = \alpha_c/\alpha_0\,,$$

$$\gamma_{rr} = \begin{cases} (1 - \nu)\,[1 + (1 - 2\sigma)\, R^2/(R + \delta)^2]\,, & \xi < 1\,; \\ (1 - \nu)\,(1 - 2\sigma)\,[R^2/(R + \delta)^2 - 1/\xi^2]\,, & \xi > 1\,; \end{cases}$$

$$\gamma_{\varphi\varphi} = \begin{cases} (1 - \nu)\,[1 + (1 - 2\sigma)\, R^2/(R + \delta)^2]\,, & \xi < 1\,; \\ (1 - \nu)\,(1 - 2\sigma)\,[1/\xi^2 + R^2/(R + \delta)^2]\,, & \xi > 1\,; \end{cases} \tag{11}$$

$$\gamma_{zz} = \begin{cases} (1 - \nu)\, 2\sigma\,[1 + (1 - 2\sigma)\, R^2/(R + \delta)^2]\,; & \xi < 1\,; \\ (1 - \nu)\, 2\sigma\,(1 - 2\sigma)\, R^2/(R + \delta)^2\,, & \xi > 1\,; \end{cases}$$

$$\gamma_{ik} = p_{ik}\, 6\,(1 - 2\sigma)\,(1 - \sigma)/E\alpha_0\,(T - T_0)\,.$$

We note at once that in the statement of problem (1)-(3) in question the length of the thread with a coating is considered as constant (as for a bar with fixed ends). Therefore it is natural that as the coating thickness δ tends to zero the stressed state does not vanish in the specimen [7].

Figure 1 presents results of calculating the deformation and stress fields in the W–B system, whose parameter distribution corresponds to the case (9). We observe maximum deformations on the surface-coating interface, and the value of the deformations decreases as the absolute value of the factor $(1 - \nu)\,(T - T_0)$ decreases or the coating thickness increases (see (10)). The stress jump on the surface on which the coating is deposited is independent of the coating thickness (see (11)). However, the value of the stresses in the fiber core ($\xi < 1$) decreases monotonically with an increase in the thickness of the deposited coating. A similar situation holds for the azimuthal and axial stresses in the coating. On the other hand, the absolute value of the radial stresses grows in the coating as the coating layer increases (see (11)).

As is evident from (11) the sign of the stresses in the fiber core or the coating is always governed by the factor $(1 - \nu)\,(T - T_0)$. There are no values of this factor at which all the stresses in the coating would be of the

TABLE 1

Type of the coating	Parameter region	Type of cracks			Note
		radial	azimuthal	axial	
Homogeneous	$(1-\nu)(T-T_0) > 0$	–	+	+	1) cracking on the boundary with the core
	$(1-\nu)(T-T_0) < 0$	+	–	–	2) scaling off on the boundary with the core
Gradient	$(1-\nu)(T-T_0) > 0$	+	+	+	1), 2)
	$(1-\nu)(T-T_0) < 0, \sigma > 1/4$	–	$\oplus$	–	3) cracking on the coating surface
	$(1-\nu)(T-T_0) < 0, \sigma < 1/4$	–	–	–	–
Multilayer	$0 < \nu < 1-a[1+1/1+\beta]$, $T-T_0 > 0$	$\oplus$	+	+	4) scaling off on the periphery, 1)
	$1-a[1+1/1+\beta] < \nu < 1-a$ $T-T_0 > 0$	$\oplus$	–	+	1), 4)
	$1-a < \nu < 1-(\beta/1+\beta)\times$ $a(1+2\lambda)$, $T-T_0 > 0$	$\oplus$	–	–	the value of the corresponding radial stresses can be made as small as desired, 4)
	$\nu > 1-(\beta/1+\beta)a(1+\lambda)$, $T-T_0 > 0$	+	–	–	2)
	$0 < \nu < 1-a[1+1/1-2\sigma]$, $T-T_0 < 0$	+	$\oplus$	–	2), 3)
	$1-a[1+1/1-2\sigma] < \nu < 1-$ $-(\beta/1+\beta)a(1-2\lambda)$, $T-T_0 < 0$	+	$\oplus$	+	1), 2), 3)
	$\nu > 1-(\beta/1+\beta)a(1+2\lambda)$, $T-T_0 < 0$	$\oplus$	+	+	1), 3)

same sign for the TEC step distribution (9). Therefore in similar systems there will always be tensile stresses, along with compression ones, in the coating layer (Fig. 1, Table 1).

Gradient Fibers. A coating that ensures a piecewise linear distribution of TEC can exemplify this system:

$$\alpha = \alpha_0 \quad \text{for} \quad 0 < r < R,$$
$$\alpha = \alpha_0 + (\alpha_c - \alpha_0)\,(r - R)/\delta \quad \text{for} \quad R \le r \le R + \delta. \tag{12}$$

Figure 2 gives the deformation and stress field for the W–BC system in which the TEC changes linearly, depending on the volume content of boron and carbon, from the fiber core TEC (α_0 at $r = R$) to some α_c (at $r = R + \delta$). Here, unlike the step distribution (9), the region of maximum deformations lies inside the coating rather than on the core interface, and the transition from expansion to compression (or vice versa) occurs smoothly:

$$
\omega = \begin{cases} \xi\,(1-\nu)\left[2\,(1-\sigma) - \dfrac{1}{3}(1-2\sigma)\left[2 - \dfrac{R}{\delta} + \dfrac{R}{\delta}\,\dfrac{R^2}{(R+\delta)^2}\right]\right], & \xi < 1\,; \\[2ex] \xi\,(1-\nu)\left[2\,(1-\sigma) - \dfrac{1}{3}(1-2\sigma)\left[2 - \dfrac{R}{\delta} + \dfrac{R}{\delta}\times\right.\right. & \\[2ex] \left.\left.\times\,\dfrac{R^2}{(R+\delta)^2}\right] - \dfrac{R}{\delta}\left[\dfrac{2}{3}\xi - 1 + \dfrac{1}{3\xi^2}\right]\right], & \xi > 1\,. \end{cases} \tag{13}
$$

The stress distribution in this system is continuous jump-free. Nevertheless, as for the step TEC, the value of the stresses in the fiber core decreases monotonically with an increase in the deposited coating thickness:

$$
\begin{aligned}
\gamma_{rr} &= \begin{cases} (1-\nu)\left[2\,(1-\sigma) - \dfrac{1}{3}(1-2\sigma)\left[2 - \dfrac{R}{\delta} + \dfrac{R}{\delta}\,\dfrac{R^2}{(R+\delta)^2}\right]\right], & \xi < 1\,; \\[2ex] (1-\nu)\left[\dfrac{2}{3}(2-\sigma) + \dfrac{R}{\delta}\left[\dfrac{2}{3}(2-\sigma) - \dfrac{1}{3}(1-2\sigma)\,\dfrac{R^2}{(R+\delta)^2}\right] - \right. & \\[2ex] \left. - \dfrac{2}{3}\,\dfrac{R\xi}{\delta}(2-\sigma) + \dfrac{1}{3}(1-2\sigma)\,\dfrac{R}{\delta}\,\dfrac{1}{\xi^2}\right], & \xi > 1\,; \end{cases} \\[3ex]
\gamma_{\varphi\varphi} &= \begin{cases} (1-\nu)\left[2\,(1-\sigma) - \dfrac{1}{3}(1-2\sigma)\left[2 - \dfrac{R}{\delta} + \dfrac{R}{\delta}\,\dfrac{R^2}{(R+\delta)^2}\right]\right], & \xi < 1\,; \\[2ex] (1-\nu)\left[2\,(1-\sigma) - \dfrac{1}{3}(1-2\sigma)\left[2 - \dfrac{R}{\delta} + \dfrac{R}{\delta}\,\dfrac{R^2}{(R+\delta)^2}\right] - \right. & \\[2ex] \left. - \dfrac{R}{\delta}\left[\dfrac{2}{3}\xi - 1 + \dfrac{1}{3\xi^2} + 2\sigma\,(\xi - 1)\right]\right], & \xi > 1\,; \end{cases}
\end{aligned} \tag{14}
$$

$$
\gamma_{zz} = \begin{cases} 2\sigma\,(1-\nu)\left[2\,(1-\sigma) - \dfrac{1}{3}(1-2\sigma)\left[2 - \dfrac{R}{\delta} + \dfrac{R}{\delta}\,\dfrac{R^2}{(R+\delta)^2}\right]\right], & \xi < 1\,; \\[2ex] 2\sigma\,(1-\nu)\left[2\,(1-\sigma) - \dfrac{1}{3}(1-2\sigma)\left[2 - \dfrac{R}{\delta} + \dfrac{R}{\delta}\times\right.\right. & \\[2ex] \left.\left.\times\,\dfrac{R^2}{(R+\delta)^2}\right] - \dfrac{R}{\delta}(\xi - 1)\right], & \xi > 1\,. \end{cases}
$$

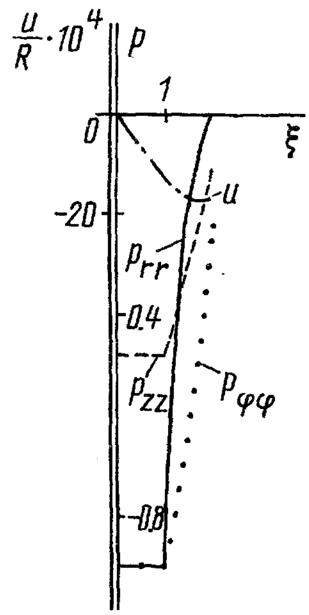

Fig. 2. Deformation and stress fields in a system with a gradient coating of W-BC: $\alpha_0 = 4.6\cdot 10^{-6}\,K^{-1}$, $\alpha_c = 1.1\cdot 10^{-6}\,K^{-1}$, $E \equiv 410$ HPa, $\sigma = 0.27$, $T - T_0 = -10^3$ K, $\delta/R = 1$.

A similar situation holds for stresses in the coating if the Poisson coefficient σ has a limited value ($\sigma < 1/4$) or the coating thickness is small ($\delta/R \lesssim 1$ (see (14)). In the alternative situation of thickly coated materials ($\sigma \geq 1/4$, $\delta/R >> 1$) that deform sufficiently strongly in the direction opposite to the applied force, the picture is somewhat different for azimuthal stresses. They change their sign. Finally, compressive azimuthal stresses are followed by tensile ones for $(1 - \nu)$ $(T - T_0) < 0$ and vice versa for $(1 - \nu)$ $(T - T_0) > 0$ (see (14)).

The sign of the stresses in system (12), similarly to the case (9), is governed by the factor $(1-\nu)$ $(T-T_0)$ but, unlike the latter, there is some region of parameters here for which all stresses appearing in the system will be, for example, compressive ones (see Fig. 2 and Table 1).

We note that the ceramic coatings of interest have a substantially lower breaking strength than compressive strength [8–10]. Therefore the first problem that we need to solve in designing these coatings is minimization (or total elimination) of dimensions of the tensile stress region. From this point of view the advantange of the gradient coatings (12) is evident. Indeed, we are able to eliminate these regions for all possible parameters ν, σ, T/T_0 of the system with the TEC step distribution (9) in the fiber. They will appear in the coating as radial scaling-off stresses for $(1 - \nu)$ $(T - T_0) < 0$ (cooling of a coating with a TEC smaller than in the core or heating of a coating with a larger TEC) or as azimuthal or axial cracking stresses for $(1 - \nu)$ $(T - T_0) > 0$ (Table 1). It is of interest that in the gradient coatings (12) cracks of a different type as compared to the systems (9) can appear for unfavorable parameters of the system: azimuthal ones but developing from the outer surface of the coating.

Multilayer Fibers. We consider multilayer fibers, using a system with a step multilayer distribution of the TEC as an example:

$$\alpha = \alpha_0 \quad \text{when} \quad 0 < r < R\,,$$
$$\alpha = \alpha_c \quad \text{when} \quad R + \Delta\,(1 + \beta)\,i < r < R + \Delta\,[(1 + \beta)\,i + 1]\,, \tag{15}$$
$$\alpha = \alpha_c + h \quad \text{when} \quad R + \Delta\,[(1 + \beta)\,i + 1] < r < R +$$
$$+ \Delta\,(1 + \beta)\,(1 + i)\,, \quad i = 0\,,\,\dots,\,N\,.$$

Omitting cumbersome expressions here for the deformation and stress fields that appear in system (15), we illustrate the obtained results in Figs. 3 and 4 for the W–SiC–B_4C system.

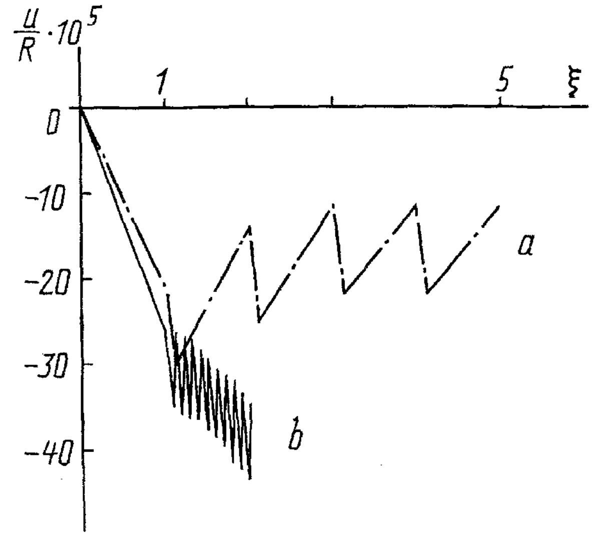

Fig. 3. Deformation field in a system with a multilayer coating of W–SiC–B_4C: $\alpha_0 = 4.6\cdot 10^{-6}\ K^{-1}$, $\alpha_c = 5.9\cdot 10^{-6}\ K^{-1}$, $\alpha_c + h = 7.1\cdot 10^{-6}\ K^{-1}$, $E \equiv 410$ HPa, $\sigma = 0.27$, $T - T_0 = -10^3$ K, $\Delta/R = 0.1$, a) $N = 3$, $\beta = 9$; b) $N = 9$, $\beta = 0.1$.

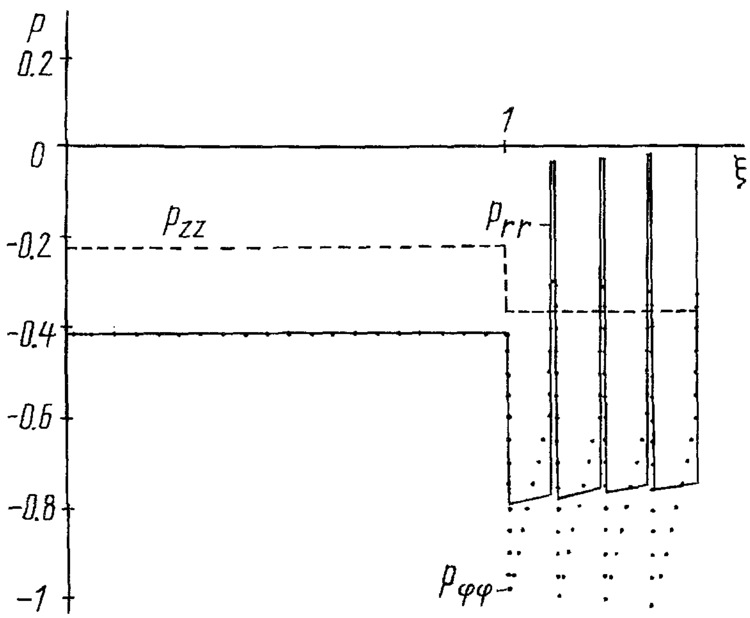

Fig. 4. Stress field in a system with a multilayer coating of W–SiC–B_4C: $\alpha_0 = 5.9\cdot 10^{-6}\ K^{-1}$, $\alpha_c = 4.6\cdot 10^{-6}\ K^{-1}$, $\alpha_c + h = 7.1\cdot 10^{-6}\ K^{-1}$, $E \equiv 410$ HPa, $\sigma = 0.27$; $T - T_0 = 10^3$ K, $\Delta/R = 0.1$, $N = 3$, $\beta = 0.1$.

1. The deformation field in the coating (15) has a sawtooth form. The frequency of deformation peaks decreases with an increase in the thickness of the coating material interlayers (with an increase in Δ or β) and their magnitude increase. Depending on the relation between the interlayer thicknesses β, we can observe a deformation maximum near the boundary with the fiber core or near the outer surface (Fig. 3).

2. The stresses change in steps in the fiber (Fig. 4). As the interlayer thickness increases, the frequency of the steps decreases but their amplitude remains constant.

3. A calculation shows that a pure mechanical increase in the number of layers in the coating does not result in a substantially altered picture of the stresses near the surface of the coated thread.

4. However, what is most important is that despite the step change in the stresses in system (15) there is some region of parameters

$$1 - \left(\frac{\beta}{1+\beta}\right) a(1 + 2\lambda) > \nu > 1 - a\left[1 + \frac{1}{1+\beta}\right], \quad T - T_0 > 0,$$
$$\nu = \alpha_c/\alpha_0, \quad a = h/\alpha_0, \quad \lambda = \Delta/R, \tag{16}$$

in which we are able to practically eliminate tensile stresses (see Fig. 4). A change in any parameter (for example, β) that results in violation of condition (16) induces tensile stresses at once (see Table 1).

Conclusion. Thus, the performed investigation shows that the experimentally observed [11] advantage of gradient and multilayer coatings over ordinary homogeneous ones with a TEC jump on the boundary with the fiber core is due to the possibility of properly selecting parameters of the system and eliminating tensile stresses. However, we need to note that these investigations took no account of adhesion between the coating layers and of the coating itself to the covered thread [8]. The results obtained are valid for systems in which the adhesion force between the layers is smaller than the strength of the coating and the substrate, which is a rather widely occurring case. Situations with ideal adhesion were analyzed in [5, 6].

NOTATION

R, thread radius; p_{ik}, stress tensor; $\alpha = \alpha(r)$, thermal expansion coefficient; r, D, radial coordinate; k, μ, moduli of hydrostatic stress and shear; u_{ik}, deformation tensor; δ, Poisson coefficient; E, Young's modulus; α_0, fiber core TEC; α_c, TEC in the coating; h, difference between the values of the TEC in the layers of a multilayer coating; Δ, thickness of the first layer in a multilayer coating; β, relative thickness of the second layer in a multilayer coating.

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